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NASH EQUILIBRIUM – A STRATEGY OF GAMES THEORY

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ABSTRACT

The paper presents an overview of the concept of Nash Equilibrium and its importance in application of games theory. Beginning with the history of Nash Equilibrium, its relevance in understanding and analyzing the outcome of strategic interactions in decision-making is elucidated with examples. Prisoners' Dilemma, Coordination game, network traffic, competition game, matching pennies, stag hunt - are some of few strategic concepts/games that exemplify the Nash Equilibrium and help in decision-making. The articles further throws light on – best response functions, dominant actions of players (which further looks at strict domination and weak domination by players), equilibrium in a single population situation and in collective decision-making situation.

INTRODUCTION

John von Neumann and Oskar Morgenstern invented the games theory in 1944. Their theory of framework was initially pertinent only under limited and exclusive circumstances. Over the past six decades this situation has changed and the framework has been generalized and deepened. These refinements have made game theory the most useful and important tool in an analyst's kit. The theory continues to be refined further by many, who are implementing the same for various analytical situations. The study of strategic interactions among different players, which produce outcomes, is known as the Game theory. These outcomes are studied with respect to preferences of the players and the outcomes. Though, they may have not expected any of the outcomes. Game theory has the its applications in various fields like – Mathematics, Social sciences, Economics, Political science, Biology and with the advancement of technologies and internet in computer science too. The games are of two types – Cooperative games and non-cooperative games.

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Cooperative game – is where groups of players / coalitions may insist on cooperative behavior. The game then is a competition between groups/ coalitions of players rather than individual players. It may again be a situation where players actually do not cooperate on their own will but are enforced to cooperate by an outside entity like a judge, police or company management etc. Non-cooperative games are those in which players are unable to make enforceable contracts. These are games where the players not necessarily cooperate, but games in which any cooperation by the players ought to be self-enforcing unlike outside entity enforcements in cooperative games. A Strategic game is a game between players with interacting decision makers model. The strategic game consists of - a set of players, a set of actions or strategies for each player and choices over the set of action profiles for each player. A wide variety of situations can be modeled as strategic games. Since, the decision-makers interact in the process we call them as players. Players here, may be companies, the actions maybe prices and the preferences may be a reflection of the company's profits; or the players can also be different political contenders, the actions would be expenditure of campaigns and their preferences would be the probability of winning the election. The choice of strategies that each player may have can be - *Pure strategies* – where the players of the game decidedly choose their moves; and *Mixed* *strategies* – where players choose one out of various strategies at random. Many-a-times the best strategy for a player to choose in a game would be mixed strategy since it provides various options. However, it is also likely that a pure strategy may be an optimal one.

Nash equilibrium

Named after Nobel Laureate, John Nash Jr, who proposed it, Nash Equilibrium is a collective set of strategies for each player who take a decision taking into account the other player(s) decision. In other case, each player is assumed to comprehend the equilibrium strategies of other players. Hence, by changing only his/her strategy unilaterally, no player has anything to gain. The players are known to be in Nash Equilibrium if each one is making a strategic decision taking into account the decisions of other players. However, Nash Equilibrium does not always mean best collective payoff for all the players involved. Some strategic games may have a single Nash Equilibrium, some may posses no Nash Equilibrium and others may have many Nash Equilibria. Nash first proposed this theory in his paper ‘ The Bargaining Problem’ in 1950, He later discussed the same in his paper ‘Equilibrium points in n-person games’ and in ‘Non-Cooperative Games’ in 1951 and ‘Two-person Cooperative Games’ in 1953. He was awarded the John von Neumann Theory Prize, in 1978, for his discovery of ‘non-cooperative equilibria’ famously now known as Nash Equilibrium.

Nash Equilibrium Examples in some games

Prisoners’ Dilemma – Is where two suspects in a crime are detained separately and kept in independent cells. Enough evidence is present to convict each one for a minor offense, but both or either cannot be convicted for a major crime unless one of them converts as an informer and provide information about the other person. In a situation where both of them stay quiet they would be convicted of a minor crime and may spend less duration in prison. The situation then can be modeled as a strategic game - by informing each one separately that if the other one provides information against him/her then he/she may be convicted for longer duration and as informer the other person may be given lesser punishment or may be set free. Both player 1 and 2 would start wondering what to do? There can be four options here – option1- both players do not confess and get punishment for minor crime and get lesser years in prison; option 2 – player 1 can confess and player 2 remains quiet, where player 1 will have no punishment and player two may get more; Option 3 – player 1 remains quiet and player 2 confesses, where player two gets no punishment and player 1 gets more; and Option 4 – where both players confess and get very less punishment.

	Confess	Deny
Confess	5,5	0,10
Deny	10,0	1,1

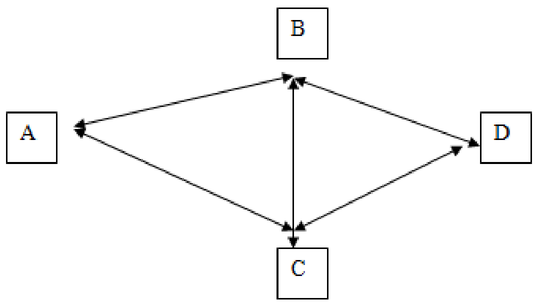
The situation where both the prisoners confess is called as a pure strategy Nash Equilibrium as if one prisoner chooses to confess than it is better for the other prisoner too to confess rather than deny and face more punishment. In an iterated or repeated prisoner’s dilemma, cooperation may be attained through trigger strategies like – tit for tat.

Coordination Game – this is a classic / symmetric two player and two strategy game with four payoffs. If both the players adopt strategy A as shown in the below matrix then the payoff is the highest and this is Nash Equilibrium. Nash Equilibrium also occurs when both the players choose strategy B, though the payoff is comparatively less than strategy A.

	Player 2 A	Player 2 B
Player 1 A	4,4	1,3
Player 1 B	3,1	3,3

This can be compared to two firms in similar situation where two technologies are available with companionable products and the firms have to choose a strategy to set the market standard. If firms choose a standard strategy – ‘A’, then both will benefit and increase sales/ profits, if they do not (B) then it would lead to few sales/ lesser profit will result. Both the strategies are Nash Equilibria.

Network Traffic - This is an extension of Nash Equilibria and draws attention to determining the flow of traffic in a network. In the figure given below for a player to reach point D from point A can take different routes – either route ABD or ACD or ABCD. For each chosen route the player would analyze the payoffs – here it would be in terms for travel time, the amount of traffic on the road and the condition of the road. The goal would be to minimize the travel time and focus is how to achieve it. Equilibrium will take place when time traveled on all the paths is approximately the same, in which case no driver/player has an incentive to change routes. If two routes have exact travel time and one does not then most of the traffic will travel through the two routes, which again would lead to Equilibrium.



Competition Game – This game can be explained with the help of a two players playing the game. Both the players have to choose an integer from ‘0’ to ‘3’ and both gain the smaller of the two numbers in points.

Player 2 ⇨ / Player 1 ⇩	0	1	2	3
0	0,0	2, -2	2, -2	2, -2
1	-2, 2	1,1	3, -1	3, -1
2	-2, 2	-1, 3	2,2	4,0
3	-2, 2	-1, 3	0, 4	3, 3

If a player chooses larger number than the other player then he/she has to give two points to him/her. A unique pure-strategy Nash Equilibrium (choosing 0,0 strategy) is observed in this game. Other than the pure-strategy Nash Equilibrium there are three more Nash Equilibria – (1,1), (2,2) and (3,3) in the game.

Matching Pennies - This is a game where two people/ players simultaneously choose to show the Head or Tail of a coin. If both show the same side – either head or tail, second person will pay first person a coin (rupee); if they show different sides of the coin, then first person will pay second person a coin. In this strategic game the payoffs are equivalent to the amounts of payoffs involved. Matching pennies game is also an example of a zero-sum game as the sum of payoffs for the two players for every choice that they make is a zero.

	H	T
H	1, -1	-1, 1
T	1, -1	1, -1

This game has mixed strategy Nash Equilibrium and no pure strategy Nash Equilibrium. Each player plays each strategy with equal probability, there by resulting in an expected payoff of zero. The game is similar to 'odds or evens' strategy and is also quite identical to – rock, paper, scissors – a three-strategy version game.

Stag Hunt

Is a game based on a discourse given by Jean-Jacques Rousseau, a philosopher, in 1755 on a group of hunters who desire to catch a stag. Each of the group of hunters have two options – they may either remain attentive to hunt the stag, or catch a hare which is comparatively easier or; if all hunters track the stag, they catch it and then share it equally. If the hunter dedicates his/her energy in catching a hare then the stag escapes and the hare belongs to one hunter only. Each hunter has a preference for a share of the stag in comparison to the hare. The strategic game similar to this is known as the Stag Hunt where, the players – are hunters, actions – are each player's set of actions; and preferences are the action profiles in which each player either chooses stag or hare. There would also be situations where when player 1 chooses stag other layer chooses stag, when player 1 chooses stag and player 2 may choose hare and vice-versa and where both the players choose hare.

	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

The Stag Hunt game involving two players has two Nash Equilibria – 'Stag, Stag' and 'Hunt, Hunt'.

Conditions effecting Nash Equilibrium

The theory of Nash Equilibrium, as discussed above, has two components – the players take action in agreement with the theory of rational choice, given their perception about other players' actions and these perceptions are correct. If every player understands the game he/she is playing and deals with incentives that match the preferences, then the deviation between the observed outcome and Nash Equilibrium can be blamed on the failure of either one or both of the components. The concept of stability is essential in all practical applications of Nash Equilibria. This is due to the fact that mixed-strategy of each player is not known completely. Conditions that guarantee that Nash Equilibrium is played include – All the players ought to do their utmost to maximize the expected payoff; players need to be flawless in execution of actions; players also need to have sufficient intelligence to infer solutions; the planned equilibrium strategy of each player is known to each other; the belief that deviation from one's own strategy will not cause deviations by any other players; and that there is common knowledge that every player meets these conditions mentioned. There are many situations where all the conditions are not met based on circumstances in which the game is played. This is again true, due to the limited conditions in which Nash Equilibrium is applied.

Like all useful theories, the theory of Nash Equilibrium is not exactly correct though we expect it to correspond to reality approximately. To check the validity of Nash Equilibrium it can be compared to other alternative theory. But, unfortunately, for many games obvious alternative theories are not available and the extent of generality that Nash Equilibrium offers no other theory offers the same. As a theoretical concept, however, Nash Equilibrium has explanatory power in economics, evolutionary biology, management games etc. John Nash's simple idea has led to fundamental changes in the fields of economics and political and other behavioral sciences. Nash Equilibrium is useful not just as an accurate interpreter of behavior of people in a game, but also when it is not, since it then, sets apart situations in which tension prevails between individual incentives and other motivations. One has to judge the theory based on the relevance and the augmentation that it provides in understanding behavior patters in a game or in reality and aids in decision – making and complex strategic environments.

REFERENCES

1. http://books.google.com/books?id=pFPHKwXro3QC&dq=nash+equilibrium&printsec
2. http://plato.stanford.edu/entries/game-theory/
3. http://www.cse.iitd.ernet.in/~rahul/cs905/
4. http://en.wikipedia.org/wiki/Nash_equilibrium
5. http://www.gametheory.net/dictionary/NashEquilibrium.html
6. http://www.economics.utoronto.ca/osborne/igt/nash.pdf
7. http://cepa.newschool.edu/het/profiles/nash.htm
8. http://www.gametheory.net/dictionary/Games/MatchingPennies.html
9. http://www.ams.org/notices/199810/milnor.pdf
10. http://kuznets.harvard.edu/~aroth/papers/HoltRoth.nash_perspective.pdf

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