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MHD BOUNDARY LAYER FLOW WITH FORCED CONVECTION PAST A NONLINEARLY STRETCHING SURFACE WITH VARIABLE TEMPERATURE AND NONLINEAR RADIATION EFFECTS

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ABSTRACT

In the present study, the steady, laminar, nonlinear hydromagnetic forced convective flow of a viscous, incompressible, electrically conducting and radiating fluid over a nonlinearly stretching surface of variable temperature with nonlinear radiation effects in the presence of a variable transverse magnetic field has been investigated. Stretching of the surface is considered in a power-law distribution form. The boundary layer equations for nonlinear flow and heat transfer are reduced to ordinary nonlinear differential equations by employing similarity transformation. The numerical solutions are found by using Fourth-Order Runge-Kutta based shooting method along with the Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions. Numerical results for velocity, temperature distribution, skin friction and rate of heat transfer on the stretching surface with nonlinear radiation effects are presented. Effect of magnetic interaction parameter and velocity exponent parameter over velocity and skin friction coefficient are analyzed. Influence of radiation parameter, local surface temperature parameter, Index of power-law variation of wall temperature and Prandtl number over temperature and dimensionless rate of heat transfer are analysed.

INTRODUCTION

The heat transfer problem due to a stretching surface that issues from a thin slit is important in a variety of engineering applications such as in the aerodynamic extrusion of plastic sheets, sheet glass, the boundary layer along material handling conveyers, hot and cold extrusions and the boundary layer along a liquid film in condensation processes. In all these cases, a study of the flow field and heat transfer can be of significant importance since the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate. A new dimension is added to the study of flow and heat transfer in a viscous fluid over a stretching surface by considering the effect of thermal radiation. Thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas. Owing to these applications, the present work chiefly deals with a problem of such kind. Tsou *et al*. (1967) studied a wide ranging analytical and experimental investigation of the flow and heat transfer characteristics of the boundary layer on a continuous moving surface. A combined analytical and experimental study of the flow and temperature fields in the boundary layer on a continuous moving surface had been carried out. The investigation includes both laminar and turbulent flow conditions. The analytical solutions provide results for the boundary layer velocity and temperature distributions and for the surface friction and heat transfer coefficients. The two-dimensional flow caused solely by a linearly stretching sheet in an otherwise quiescent incompressible fluid which has a very simple closed form exponential solution was established by Crane (1970). Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet was studied by Chiam (1977). Grubka and Bobba (1985) carried a detailed analysis of characteristics of a continuous, stretching surface with variable temperature. Chiam (1995) reported solution for steady hydromagnetic flow

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over a surface stretching with a power-law velocity with the distance along the surface. Unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium was studied by Chamkha (1998). Heat transfer over a stretching surface with variable surface heat flux was analysed by Elbashbeshy (1998). Chen (2000) investigated the effect of heat transfer characteristic of a non-isothermal surface moving parallel to a free stream. The effect of steady nonlinear MHD flow and heat transfer over a surface stretching with a power-law velocity was investigated by Anjali Devi and Thiagarajan. (2003). Chen (2004) analysed the effect of heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration. Flow and Heat transfer of viscous fluids saturated in porous media over a permeable non-isothermal stretching sheet was analyzed by Chung (2006).

Prasad *et al*. (2009) carried a detailed analysis of MHD power-law fluid flow and heat transfer over a non-isothermal stretching sheet. Radiative MHD flow over a non-isothermal stretching sheet in a porous medium was studied by Paresh and Srivastava (2010). In most of the above mentioned studies, the radiation term appears in linear form. Elbashbeshy (2000) analysed the radiation effect on heat transfer over a stretching surface by taking into account of the full form of radiation term. Recently, Swati Mukhopadhyay (2011) investigated the effect of boundary layer flow and heat transfer over a porous moving plate in the presence of thermal radiation, taking into account of the full form of radiation term. Upto author's knowledge, no attempt has been made so far to study the effect of MHD boundary layer flow with forced convection past a nonlinearly stretching surface with variable temperature and nonlinear radiation effects.

Formulation of the Problem

Let us suppose a steady, laminar, two-dimensional nonlinear MHD boundary layer flow with heat transfer of a viscous, incompressible, electrically conducting and radiating fluid over a nonlinearly stretching surface with variable temperature in the presence of a transverse variable magnetic field and nonlinear radiation effects. The fluid is assumed to be a gray, emitting and absorbing but non-scattering medium.

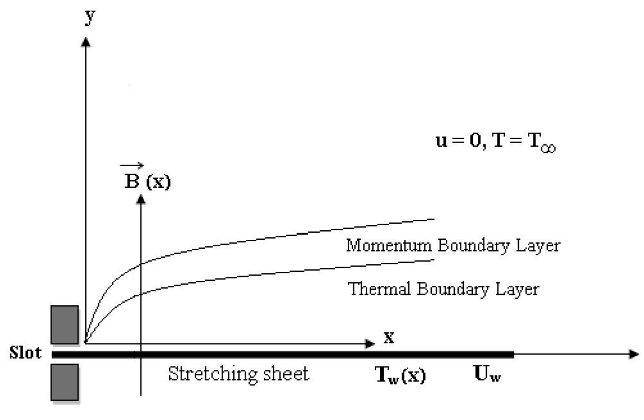

Fig. 1. Schematic diagram of the problem

Assume that the speed of a point on the surface is considered in a power-law distribution form and the boundary layer approximations are applicable. Cartesian co-ordinate system is chosen and the velocity components corresponding to x and y directions are respectively denoted by u and v. The governing boundary layer equations of the problem are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\left[\frac{\sigma B^2(x)}{\rho}\right]u \tag{2}$$

where $B(x)=B_o x^{\frac{(m-1)}{2}}$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

together with relevant boundary conditions

$$\begin{aligned} &u=u_w=u_o x^m,\quad v=0,\ T=T_w(x) \quad at\ y=0 \quad (u_o>0)\\ &u=0,\quad T=T_\infty \quad as\ y\to\infty \end{aligned} \tag{4}$$

where u_w is the velocity of the stretching surface, u_o is a dimensional constant, m is the index of power-law velocity which is a constant, ν is kinematic viscosity, σ is the electrical conductivity, ρ is the density, C_p is the specific heat at constant pressure, K is the thermal conductivity, T is the temperature of the fluid, $T_w(x)$ is the wall temperature, q_r is the radiative heat flux and T_∞ is the temperature of the fluid at infinity. It is assumed that the induced magnetic field is negligible for induced magnetic field $\vec{h}$ << applied magnetic field and the external electrical field is zero as curl $\vec{E}$ = 0 and div $\vec{E}$ = 0 when the electric field due to polarization of charges is negligible. Viscous and Joule's dissipation effects are neglected. The radiative heat flux in the x direction is considered to be negligible in comparison to the flux in the y direction. The radiative heat flux term is simplified by using the Rosseland diffusion approximation and accordingly

$$q_r=-\frac{16\sigma^* T^3}{3\alpha^*}\frac{\partial T}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient. Introducing the usual similarity transformation (Ali (1995)),

$$\eta(x,y)=y\sqrt{\frac{m+1}{2}}\sqrt{\frac{u_o x^{m-1}}{\nu}} \tag{6}$$

$$\psi(x,y)=\sqrt{\frac{2}{m+1}}\sqrt{\nu u_o x^{m+1}}\,f(\eta) \tag{7}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \theta_{wx} = \frac{T_w(x)}{T_\infty} \tag{8}$$

where θ_{wx} is the local surface temperature parameter.
Velocity components are obtained as

$$u = u_w f'(\eta) \tag{9}$$

$$v = -\sqrt{\frac{m+1}{2}}\sqrt{\frac{\nu u_w}{x}}\left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta)\right] \tag{10}$$

Defining $T_w(x) = T_\infty + bx^n$ where b is a dimensional constant and n is the index of power-law variation of wall temperature which is a constant, equations (2) and (3) can be written as

$$f''' + ff'' - \frac{2m}{m+1}f'^2 - M^2 f' = 0 \tag{11}$$

$$\left\{1 + \frac{4}{3R^*}(1+(\theta_{wx}-1)\theta)^3\right\}\theta'' + \frac{4}{R^*}(1+(\theta_{wx}-1)\theta)^2(\theta_{wx}-1)\theta'^2 + \Pr\left[f\theta' - \frac{(2n)}{(m+1)}f'\theta\right] = 0 \tag{12}$$

where $R^* = \dfrac{K\alpha^*}{4\sigma^* T_\infty^3}$ is the radiation parameter

$M = \sqrt{\dfrac{2\sigma B_0^2}{\rho u_o (m+1)}}$ is the magnetic interaction parameter

$Pr = \dfrac{\mu C_p}{K}$ is the Prandtl number

Associated boundary conditions are

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \\ &f'(\infty) = 0, \quad \theta(\infty) = 0 \end{aligned} \tag{13}$$

Numerical solution of the problem

Equations (11) and (12) are nonlinear ordinary differential equations which constitute the nonlinear boundary value problem. As no prescribed method is available to solve nonlinear boundary value problem, it has to be reduced to an initial value problem. This is done by using shooting method. Using Fourth-Order Runge - Kutta based shooting method with the utilization of Nachtsheim- Swigert iteration scheme for satisfaction of the asymptotic boundary conditions, numerical solution for the velocity and temperature are obtained for different values of the physical parameters such as magnetic interaction parameter, velocity exponent parameter, radiation parameter, Prandtl number and index power-law variation of wall temperature. It is essential to note that the success of this method depends greatly on the initial guess, which is made for the values $f''(0)$ and $\theta'(0)$ to initiate the shooting process and also such a guess is not to be made arbitrarily but prudently.

RESULTS AND DISCUSSION

The numerical solution of hydromagnetic boundary layer flow and heat transfer over a nonlinearly stretching surface with variable temperature has been obtained by using Fourth-Order Runge-Kutta shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of the asymptotic boundary conditions by fixing several values for the physical parameters. Numerical values are depicted graphically by means of figures for transverse velocity $f'(\eta)$, temperature distribution $\theta(\eta)$ for several sets of values of magnetic interaction parameter *M*, velocity exponent parameter *m*, radiation parameter *R**, the index of power law variation of wall temperature *n* and Prandtl number *Pr*. In the absence of radiation parameter, the results are identical to those of Anjali Devi and Thiagarajan (2003) which are justified through Figs.2 and 3.

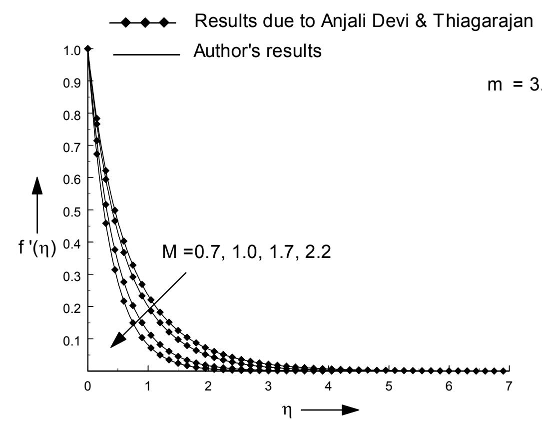

Fig.2 Velocity profiles for different M

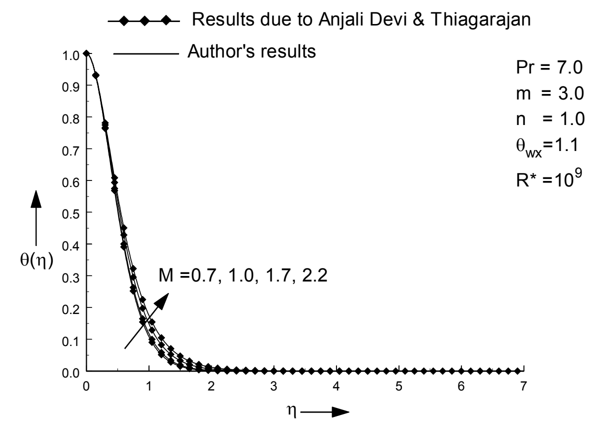

Fig.3 Temperature profiles for different M

The dimensionless velocity $f'(\eta)$ for different values of velocity exponent parameter *m* is plotted through Fig.4. It is noted that as velocity exponent parameter *m* increases, velocity $f'(\eta)$ decreases. The effect of magnetic field over the dimensionless velocity $f'(\eta)$ for a nonlinearly stretching surface is shown with the help of Fig.5. Increasing magnetic interaction parameter *M* is to reduce the velocity elucidating the fact that the effect of magnetic field is to decrease the velocity.

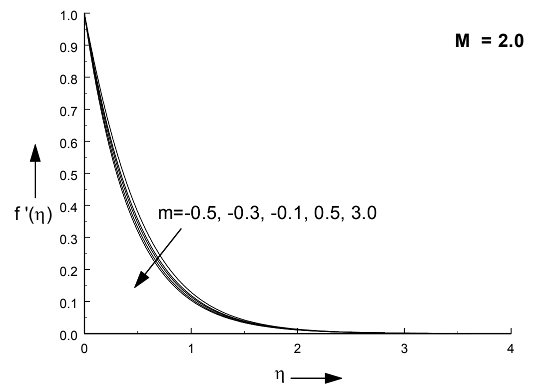

Fig.4 Velocity profiles for different m

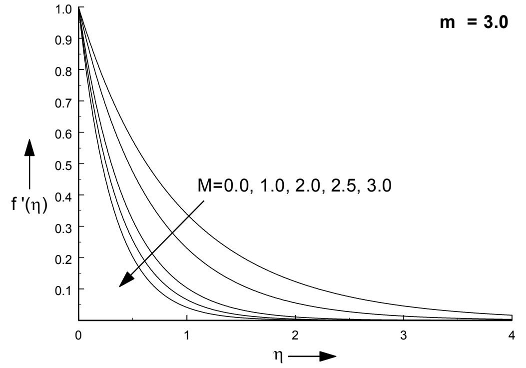

Fig.5 Velocity profiles for different M

Figure 6 illustrates the effect of magnetic field over the dimensionless temperature $\theta(\eta)$ for a nonlinearly stretching surface. Increasing magnetic interaction parameter *M* is to increase the temperature.

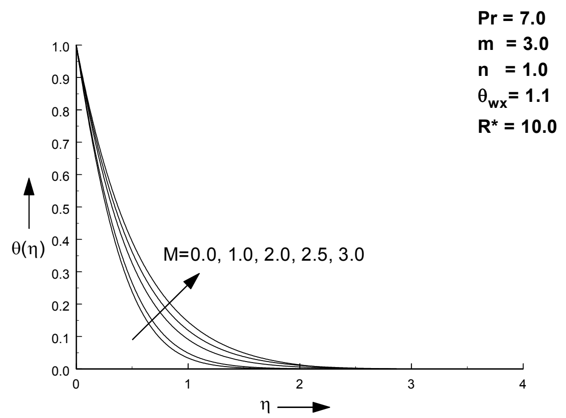

Fig.6 Temperature profiles for different M

Prandtl number variation over the dimensionless temperature distribution when the surface is nonlinearly stretching is elucidated through Fig.7. As Prandtl number *Pr* increases temperature $\theta(\eta)$ decreases, illustrating the fact the effect of Prandtl number is to decrease the temperature in the presence of magnetic field. Furthermore, the effect of Prandtl number is to reduce the thickness of thermal boundary layer.

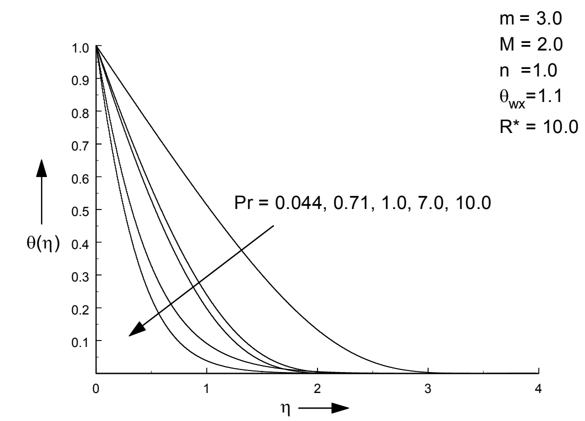

Fig.7 Temperature profiles for different Pr

Influence of *n* over the dimensionless temperature is displayed through Fig.8. It is inferred that the effect of *n* is to reduce the dimensionless temperature.

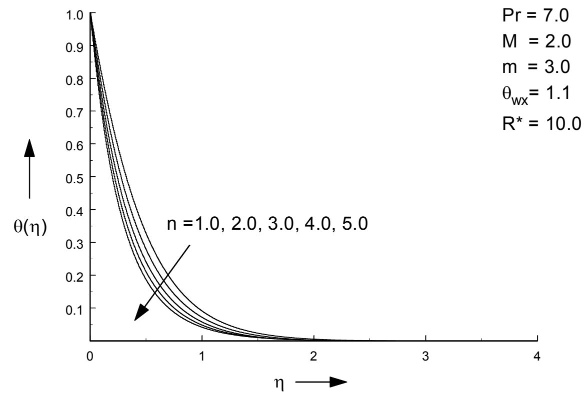

Fig.8 Temperature profiles for various n

Figure 9 illustrates the effect of radiation parameter *R** over the dimensionless temperature $\theta(\eta)$, for a nonlinear stretching surface. It is observed that the effect of radiation parameter is to reduce the temperature, elucidating the fact that the thermal boundary layer thickness decreases as *R** increases.

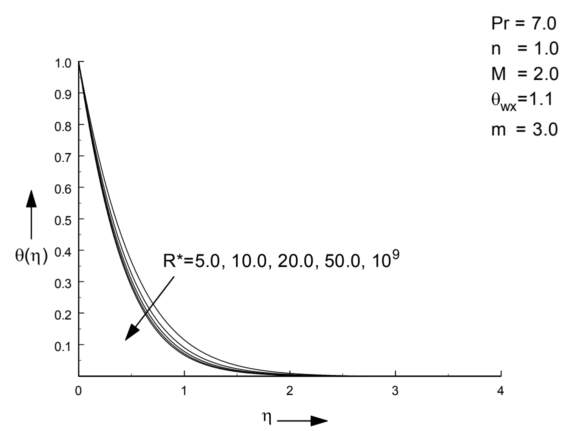

Fig.9 Temperature profiles for various R*

The magnetic interaction parameter on skin friction coefficient $f''(0)$ for different values of velocity exponent parameter for a nonlinearly stretching surface is shown through Figure 10. It is seen that the skin friction $f''(0)$ decreases with increase of velocity exponent parameter *m* and also decreases for increasing *M*.

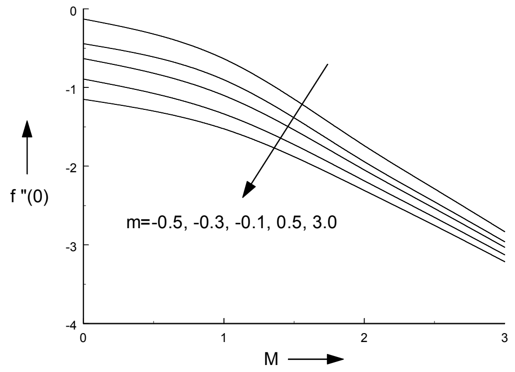

Fig.10 Skin friction coefficient for different m

Figure 11 depicts the dimensionless rate of heat transfer $\theta'(0)$ against values of magnetic interaction parameter *M* for different values of *n*. It is seen that the rate of heat transfer $\theta'(0)$ decreases with increase of *n* and increases when magnetic interaction parameter *M* increases.

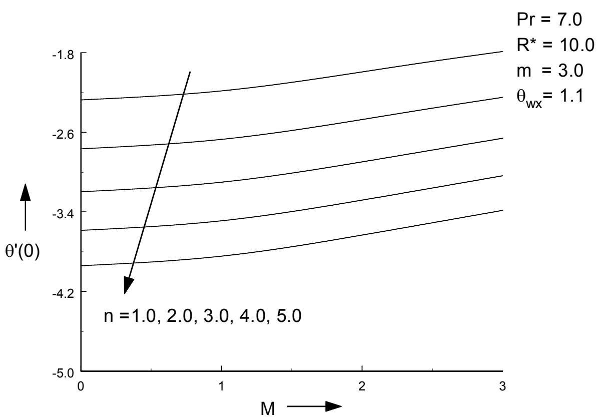

Fig.11 Dimensionless rate of heat transfer for different n

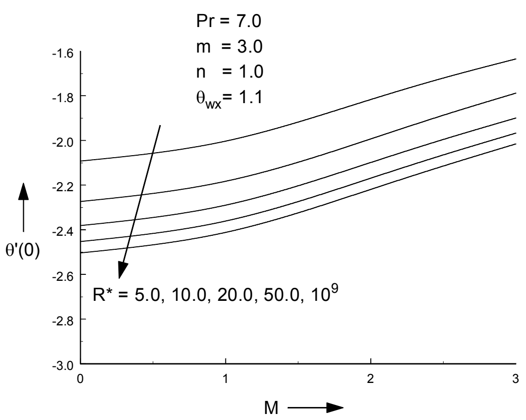

Fig.12 Dimensionless rate of heat transfer for different R*

Figure 12 displays the dimensionless rate of heat transfer $\theta'(0)$ against magnetic interaction parameter *M* for different $R*$ when the surface is nonlinearly stretching. It is seen that the rate of heat transfer $\theta'(0)$ decreases with increase of radiation parameter $R*$ and increases with the increasing magnetic interaction parameter *M*.

Conclusion

In this work the problem of nonlinear hydromagnetic forced convection flow with nonlinear radiation over a nonlinearly stretching surface with variable temperature is investigated. The results are presented for various values of the physical parameters including the radiation parameter, velocity exponent parameter, magnetic interaction parameter, index of power-law variation of wall temperature and Prandtl number. In the absence of radiation parameter, the results are identical to those of Anjali Devi and Thiagarajan (2003).

- It is found that the effect of magnetic field is to decelerate the velocity and increase the temperature.
- The effect of radiation parameter is to reduce the temperature.
- The thermal boundary layer thickness decreases sharply with increasing Prandtl number.
- The effect of magnetic field and increasing velocity exponent parameter both have the same individual effect over skin friction coefficient so as to reduce it.
- It is observed that as n increases the dimensionless rate of heat transfer decreases whereas magnetic interaction parameter increases, the non-dimensional rate of heat transfer also increases.
- It is depicted that as Radiation parameter increases the non-dimensional rate of heat transfer decreases whereas for increasing values of magnetic interaction parameter, the rate of heat transfer increases.
- The rate of heat transfer increases for increasing magnetic interaction parameter.

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