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MINIMUM RISK ESTIMATION OF SCALE PARAMETER OF LENGTH BIASED MAXWELL DISTRIBUTION WITH CENSORING UNDER ENTROPY LOSS FUNCTIONS

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ABSTRACT

In this paper, we have discussed Length biased distribution arise when the probability of inclusion of population unit in sample is related to the value of the variable measured. For example in textile sampling modeling, Cox(1969). In this paper the appropriateness of a length biased Maxwell distribution has been given. The minimum risk estimator of its scale has been obtained under entropy loss function with the help of a type II censored sample. We have obtained that the MELO estimator is nothing but the usual maximum likelihood estimator.

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INTRODUCTION

In many a situation experimenter do not work with truly random sample from the population, in which they are interested, either by design or because of the fact that in some situations it becomes impossible to have random sample from the targeted population. However, if the observations do not have an equal probability of entering the sample, the resulting sampled distribution does not follow the original distribution. Statistical models that incorporate this restriction are called weighted models. The concept of weighted distribution can be found in a paper of R.A. Fisher "The study of effect of methods of ascertainment upon estimation of frequency" in1934,while the length-biased sampling was developed by Cox (1962), Patil, et al. (1986) presented a list of the most common forms of the weighted function useful in scientific and statistical literature as well as some basic theorems for weighted distribution and size- biased as the special case they arrived at the conclusion that the length –biased version of some mixture distributions. Several authors such as Jain, et al. (1989), Gupta and Kirmani (1990), George (2002) etc, studied the various length- biased distribution and expressed in relation with these of original distributions. Maxwell distribution plays an important role in life testing and reliability theory. Let X be a random variable having the Maxwell distribution, with pdf given by:

$$f(x;\theta) = \frac{4}{\sqrt{\pi}} \cdot \frac{x^2}{\theta^{\frac{3}{2}}} e^{\frac{-x^2}{\theta}} \qquad x>0,\ \theta>0 \tag{1.1}$$

where θ is scale parameter. For the Maxwell distribution, the raw moments are given by:

$$\mu_r' = \frac{2}{\sqrt{\pi}} \theta^{\frac{r}{2}} \Gamma\left(\frac{r+3}{2}\right), \tag{1.2}$$

Thus the mean and variance are obtained as

$$\mu_1' = E(x) = 2\sqrt{\frac{\theta}{\pi}}; \tag{1.3}$$

and

$$\mu_2 = V(x) = \frac{\theta(3\pi-8)}{2\pi}; \tag{1.4}$$

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Let T be a random variable having the length biased Maxwell distribution, whose pdf is

given by:

$$f(t;\theta) = \frac{2}{\theta^2} t^3 e^{\frac{-t^2}{\theta}};\ t,\theta>0 \tag{1.5}$$

The density (1.5) can be obtained by the definition of the length-biased distributions

$$\text{Given by:-} f(t;\theta) = \frac{tg(t)}{E(t)} \tag{1.6}$$

Where

$f(t;\theta)$ follows Maxwell distribution as given in (1.1).

Let us suppose that n items are put to test for their life times and the experiment is terminated when r(< n) items have

failed, If $t_1,\ldots,t_r$. denote the first r observations having common pdf as given in (1.5), then the joint pdf is given by:-

$$f(t;\theta) = \frac{n!}{r!(n-r)!}\frac{2^r}{\theta^{2r}}\Pi_{i=1}^r t_i^3\, e^{\frac{-z}{\theta}};\quad t,\theta>0; \tag{1.7}$$

where

$$z = [\textstyle\sum_{i=i}^r t_i^2 + (n-r)\ t_r^2]; \tag{1.8}$$

The maximum likelihood estimator (MLE)$\hat{\theta}$ of θ may be obtained as;

$$\hat{\theta}=\frac{z}{2r}; \tag{1.9}$$

The pdf of $\hat{\theta}$ is given by

$$f(\hat{\theta}) = \frac{1}{\Gamma(2r)}\left(\frac{2r}{\theta}\right)^{2r} \hat{\theta}^{2r-1}\, e^{\frac{-2r\hat{\theta}}{\theta}};\quad \hat{\theta}>0 \tag{1.10}$$

The minimum risk (melo) estimator under entropy loss function

Entropy loss function is given as:

$$L(\Delta) = b(\Delta - log\Delta - 1), where\ \Delta = \frac{\hat{\theta}}{\theta} \tag{2.1}$$

The expected loss (Risk) function of MLE$\hat{\theta}$under Entropy loss $L(\Delta)$,denoted by$R_{e(\hat{\theta})}$, is given by

$$R_{e(\hat{\theta})} = \int_0^\infty b\left(\frac{\hat{\theta}}{\theta} - log\frac{\hat{\theta}}{\theta} - 1\right)f(\hat{\theta})d\hat{\theta}$$

$$=b\left[\frac{1}{\theta}\int_0^\infty \frac{1}{(2r)!}\left(\frac{2r(\hat{\theta})}{\theta}\right)^{2r} e^{-\left(\frac{2r(\hat{\theta})}{\theta}\right)}d\hat{\theta} - \int_0^\infty log_e(\hat{\theta})f(\hat{\theta})d\hat{\theta} + (log_e\theta - 1)\int_0^\infty f(\hat{\theta})d\hat{\theta}\right]$$

$$= b[1 - \int_0^\infty log_e(\hat{\theta})f(\hat{\theta})d\hat{\theta} + (log_e\theta - 1)]$$

$$= b\left[log_e\theta - E_\theta\left(log_e(\hat{\theta})\right)\right] \tag{2.2}$$

Let us define $\theta^* = M\hat{\theta}$(2.3)

Risk function of θ^*under Entropy loss function is given as:

$$R_e(\theta^*) = \int_0^\infty b\left(\frac{\theta^*}{\theta} - log\frac{\theta^*}{\theta} - 1\right)f(\hat{\theta})d\hat{\theta}$$

$$=b[M - \int_0^\infty log_e(\hat{\theta})f(\hat{\theta})d\hat{\theta} - log_eM + log_e\theta - 1]$$

$$= b\left[M - 1 - log_eM + log_e\theta - E\left(log_e(\hat{\theta})\right)\right] \tag{2.4}$$

Now in order to M lead to minimum of $R_e(\theta^*)$, we must have

$$\frac{dR_e(\theta^*)}{dM} = b - \frac{b}{M} = 0$$

$$M = 1 \tag{2.5}$$

$$\text{Since } \frac{d^2R_e(\theta^*)}{dM^2} = \frac{b}{M^2} > 0 \tag{2.6}$$

Now the minimum expected loss is given by

$$R_e(\theta^*) = b[log_e\theta - E(log_e(\hat{\theta}))] \tag{2.7}$$

Let us define the Relative efficiency of θ^* with respect to $\hat{\theta}$ as

$$\text{Rel. eff.}(\theta^* / \hat{\theta}) = \frac{R_e(\hat{\theta})}{R_e(\theta^*)} = 1 \tag{2.8}$$

Conclusion

From equation (2.8) it is clear that $\theta^*$is itself MELO estimator of θ and is best in its class when the entropy loss is considered as the criterion of judging the precision of estimator of θ in length biased Maxwell distribution.

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