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MORE ON VAGUE SETS

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ABSTRACT

A vague set is the set of objects, characterized by a truth membership function and a false membership function whose values is a continuous subinterval of [0,1] .The notion of the difference on vague sets is defined and studies various properties on them.

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INTRODUCTION

In classical set theory, the membership of elements in a set assessed in binary terms according to bivalent condition-an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. In 1965 Zadeh's classical concept of fuzzy sets is a strong mathematical tool to deal with the vagueness. The membership in a fuzzy set is not a matter of affirmation or denial but rather a matter of a degree. In 1986, Attanasov introduced the notation of hesitation part by defining a non-membership function γ_A associated with the membership function μ_A for the first time which led to the definition of intuitionistic fuzzy sets. Gau and Buehrev (1993) introduced the concept of vague sets and it was shown that a vague set is more expressive in capturing vagueness data. A vague set is the set of objects, each of which has a grade of membership whose value is continuous subinterval of [0, 1]. Such a set is characterized by a truth-membership function and a false membership function. Hence, an interval vague set is one of the higher order fuzzy sets and is being applied in various fields. The notion of true membership function, false membership function and uncertainly function in interval vague sets describes the objective world more realistically and practically. IVS reflects people's understanding in three aspects comprehensively: support degree, negative degree and uncertainty degree. In this paper some basic concepts related to fuzzy sets, intuitionistics fuzzy sets and vague sets are discussed. Finally the notion of difference on vague sets is defined and various properties on vague sets are established.

2. VAGUE SETS & INTUITIONISTIC FUZZY SETS

In this section, we discuss some basic concepts related to vague set & intuitionistics fuzzy set and also the notion of the difference on the vague sets is introduced

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2.1 BASICS

Let X be a classical sets of objects, called the universe of discourse, where an element of X is defined by x.

DEFINITION-2.1.1 FUZZYSETS

A fuzzy set $A = \{< x, \mu_A(x) > : \ x \in X\}$ in a universe of discourse X is characterized by a membership function μ_A as $\mu_A: X \to [0,1]$.

For any two fuzzy sets A and B on X, we have

$A = B \ iff \ \mu_A(x) = \mu_B(x), for\ all\ x \in X$
$A \subseteq B \ iff \ \mu_A(x) \leq \mu_B(x), for\ all\ x \in X$
$B \supseteq A \ iff \ A \subseteq B$

$\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}, for\ all\ x \in X$
$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, for\ all\ x \in X$

and the complement μ_{A^-} of the fuzzy sets A with respect to universal set X is defined as

$\mu_{A^-}(x) = 1 - \mu_A(x), for\ all\ x \in X$

Fuzzy sets require that each element of the universal set be assigned a particular real number. We may be able to identify appropriate membership function $only\ approximately$.

DEFINITION-2.1.2 INTUITIONISTIC FUZZY SETS (IFS)

Let a set X be fixed. An intuitionstics fuzzy sets or a IFS on X is an object having the form**:** $A = \{< X, \mu_A(x), \gamma_A(x) > : x \in X\}$, Where the function $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$define the degree of membership and degree of non membership function respectively of element $x \in X$ to the set A and for every$\in X$,$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.
If A & B are two IFS on the set X, then .

$A \subseteq B \ \ iff \ \ for\ all\ \ x \in X, \mu_A(x) \leq \mu_B(x)\ and\ \gamma_A(x) \geq \gamma_B(x)$
$A \subseteq B \ \ iff \ \ B \supseteq A$
$A = B \ \ iff \ \ for\ all\ \ x \in X,\ \mu_A(x) = \mu_B(x)\ and\ \gamma_A(x) = \gamma_B(x)$.
$A^- = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$
$A \cap B = \{< x, min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) >: x \in X\}$
$A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min((\gamma_A(x), \gamma_B(x)) >: x \in X\}$
$\mathrm{A} + B = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x).\gamma_B(x) >: x \in X\}$
$A - B = \{< x, \mu_A(x).\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x) >: x \in X\}$
$C(A) = \{< x, K, L >: x \in X\}$, where
$K = \max \mu_A(x)\ , x \in X\ and\ L = \min\ \gamma_A(x)\ , x \in X$
$I(A) = \{< x, k.l >: x \in X\}$, where $k = \min \mu_A(x)\ , x \in X\ and\ l = \max\ \gamma_A(x)\ , x \in X$
Obviously every fuzzy set has the form $\{< x,\ \mu_A(x), \mu_{A^c}(x) >: x \in X\}$

Hence IFSs are not fuzzy sets but fuzzy sets are intuitionstics fuzzy sets. There are many situations, where fuzzy set theory cannot be suitable applied but IFS theory can be to make a more fair analysis .Till now, exact determination of membership values of the elements in fuzzy sets is impossible. Also, there are no universal formulae to determine the membership values. Hence all the problems of our real life situation cannot be classified into a single or at worst into infinite number of classes. Even for a particular situation, the membership Value cannot always be determined, due to insufficient of available information, besides the presence of vagueness in the information. Similarly, while determining the non membership values same problem arises. A part of such estimation naturally remains in deterministic. In fuzzy set theory the in deterministic part is zero, by assumption that part of the degree of membership is determinism. But in real life situation, it is not always so, and for such environment there is a need for the IFS theory without any confusion.

DEFINITION-2.1.3 (VAGUE SETS)

A vague set A in a universe of discourse X is characterized by a true membership function, t_A and a false membership function $f_A$as follow: $t_A: x - [0,1], f_A: x - [0,1] and\ 0 \leq t_A(x) + f_A(x) \leq 1$ Where t_A is a lower bound on the grade of membership of x derived from the evidence for x and f_A Is a lower bound on the grade of membership of the negation of x derived from the evidence against x. The basic approach on vague sets which includes complete containment, equal, union, intersection and so on from [6].

DEFINITION-2.1.4 Containment: If A & B are two vague set on a set X. Then a vague set A is contained in other vague set B, $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x), for\ all\ x \epsilon X$

DEFINITION-2.1.5 Equal

A=B if and only if $t_A(x) = t_B(x)$ and $1 - f_A(x) = 1 - f_B(x)$,$for\ all\ x\epsilon X$

DEFINITION-2.1.6 Union

$A \cup B$ if and only if $t_{A\cup B}(x) = \max(t_A(x), t_b(x))$, $for\ all\ x\epsilon X$
$1 - f_{A\cup B}(x) = \max(1 - f_A(x), 1 - f_B(x)) = \min(f_A(x), f_B(x))$, $for\ all\ x\epsilon X$

DEFINITION-2.1.7 Intersection

$A \cap B$ if and only if $t_{A\cap B}(x) = \min(t_A(x), t_B(x))$ $for\ all\ x\epsilon X$ $1 - f_{A\cap B}(x) = min(t_A(x), t_B(x))$
$= \min(1 - f_A(x), 1 - f_B(x)) = 1 - \max(f_A, f_B)$, $for\ all\ x\epsilon X$

DEFINITION-2.1.8 Complement

The complement of vague set A is denoted by A' which is defined as

$t_{A'}(x) = f_A(x)\ and\ 1 - f_{A'}(X) = 1 - t_A(x), for\ all\ x\epsilon X$

Note: A vague set is empty if and only if its truth membership function and false membership function are identically zero and one.

Here, we define the notion of difference on a vague set as follows;

DEFINITION-2.1.9 Difference

The difference of two vague sets A & B is denoted by $A\backslash B$ whose true membership function and false membership function defined as

$t_{A\backslash B}(x) = t_A(x) \wedge t_{B'}(x) = \min(t_A(x), f_B(x))$, $for\ all\ x\epsilon X$

$1 - f_{A\backslash B}(x) = (1 - f_A(x)) \wedge (1 - f_{B'}(x))$
$= \min(1 - f_A(x), 1 - t_B(x))$, $for\ all\ x\epsilon X$

We can see that the difference between vague sets and IFS is due to the definition of membership intervals we have $[t_A(x), 1 - f_A(x)]$ for x in vague set a but $< \mu_A(x), \gamma_A(x) >$for x in IFS A. Hence the semantics of μ_A is same as with t_A and γ_A is same as withf_A . However, the boundary $1 - f_A$ is able to indicate the possible existence of a data value. But, the hesitation part $\Pi_A = 1 - \mu_A - \gamma_A$ corresponds to the intuition of representing vague data. Hence the notion of IFS and vague sets are regarded as equivalent in the sense that IFS is isomorphic to vague sets.

3. PROPERTIES ON VAGUE SET

In this section some properties are discuss which has been established on[6] .Here we extend and established some more properties on vague sets with the operation of union, intersection complement and difference etc.

3.1: Results on vague sets

The following results which has been deduced in [6].

If A, B and C are any three vague sets on X, then

- $A \cup B$ is the smallest vague set containing both A & B i.e A⊆ A U B and $B \subseteq A \cup B$
- $A \cap B$ is the largest set contained in A and B.
- Commutative: AUB =BUA
- Associative: AU (BUC) = (AUB) UC and $A \cap (B \cap C) = (A \cap B) \cap C$
- Idempotency: $A \cup A = A$ and$A \cap A = A$
- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- Vii. $A \cap \emptyset = \emptyset, A \cup X = X, where\ \emptyset = null\ vague\ set = [0,0]\ and\ X = [1,1]$
- Identity: $A \cup \emptyset = A\ and\ A \cup X = A, where\ \emptyset = [0,0]\ and\ X = [1,1]$
- Absorption: $A \cup (A \cap B) = A\ and\ A \cap (A \cup B) = A$
- Involution: $(A')' = A$

Here we established some results on vague sets

THEOREM: 3.1 Demerger's law

If A, B and C are vague sets on X, then

- $(A \cup B)' = A' \cap B'\ and (\ A \cap B)' = A' \cup B'$
- $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$ and $A\backslash(B \cap C) = (A\backslash B) \cup (A\backslash C)$

Proof

$t_{(A\cup B)}(x) = \max(t_A(x), t_B(x))\ , for\ all\ x\epsilon X$
$1 - f_{A\cup B}(x) = \max\left(1 - f_A(x), 1 - f_B(x)\right)$
$= 1 - \min\left(f_A(x), f_B(x)\right)\ , for\ all\ x\epsilon X \iff t_{(A\cup B)'}(x) = \min(f_A(x), f_B(x)) = t_{A'\cap B'}(x)\ , for\ all\ x\epsilon X$
$1 - f_{(A\cup B)'}(x) = 1 - \max\left(t_A(x), t_B(x)\right) = \min\left(1 - t_A(x), 1 - t_B(x)\right) = 1 - f_{A'\cap B'}(x), for\ all\ x\epsilon X$
Hence $(A \cup B)' = A' \cap B'$

Similarly, second one also can be proved.

$t_{A\backslash(B\cup C)}(x) = \min\left(t_A(x), f_{B\cup C}(x)\right)$
$= \min\left(t_A(x), min\left(f_B(x), f_C(x)\right)\right)$
$= \min(\min(\ t_A(x), f_B(x)), \min(\ t_A(x), f_C(x)))$
$= \min\left(t_{A\backslash B}(x), t_{A\backslash C}(x)\right) = t_{(A\backslash B)\cap(A\backslash C)}(x)\ , for\ all\ x\epsilon X$ and
$1 - f_{A\backslash(B\cup C)}(x) = \min(1 - f_A(x), 1 - t_{B\cup C}(x))$
$= \min(1 - f_A(x), \min(1 - t_B(x), 1 - t_C(x))$
$= \min(\min(\ 1 - f_A(x), 1 - t_B(x)), \min(1 - f_A(x), 1 - t_C(x)\ ))$
$= \min(1 - f_{A\backslash B}(x), 1 - f_{A\backslash C}(x))$
$= 1 - f_{(A\backslash B)\cap(A\backslash C)}(x)\ , for\ all\ x\epsilon X$

Hence the result $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$.

Similarly second one can be proved.

Here we furnish an example to established the above theorem

Example-3.1

If one of the element in the vague sets A,B and C are

$A = [0.5\ , 0.8] \Rightarrow\ t_A = 0.5\ , f_A = 0.2$
$B = [0.4\ , 0.7] \Rightarrow\ t_B = 0.4\ , f_B = 0.3$
$C = [0.3\ , 0.6] \Rightarrow\ t_C = 0.3\ , f_C = 0.4$

Now

$t_{A\backslash(B\cup C)} = [0.5, 0.8] \backslash[0.4, 0.7]$
$= \min(0.5, 0.3) = 0.3$
$t_{(A\backslash B)\cap(A\backslash C)} = \min\left(t_{(A\backslash B)}, t_{(A\backslash C)}\right)$
$= \min\left(min(t_{A,} f_B), \min\left(t_{A,} f_C\right)\right) = \min(0.3, 0.4) = 0.3$
$1 - f_{A\backslash(B\cup C)} = \min\left(1 - f_A\ , 1 - t_{B\cup C}\right)$
$= \min(1 - f_A), \min\left(1 - t_B, 1 - t_C\right)$
$= \min(0.8\ , \min(0.6, 0.7))$
$= \min(0.8, 0.6) = 0.6$
$1 - f_{A\backslash B}\ 1 - f_{A\backslash C} = \min\left(1 - f_{A\backslash B,} 1 - f_{A\backslash C)}\right)$
$= \min\{\min(1 - f_A, 1 - t_B), \min(1 - f_A, 1 - t_C)\}$
$= \min\{\min(0.8, 0.6), \min(0.8, 0.7)\}$
$= \min(0.6, 0.7) = 0.6$

Hence the results established.

THEOREM: 3.2

Let A , B and C are three vague sets on X, then

- $A\backslash B = A \cap B' = A\backslash(A \cap B) = B'\backslash A'$
- $(A \cap B)\backslash C = A \cap (B\backslash C)$
- $A\backslash(B \cup C) = (A\backslash B)\backslash C$

Proof:

Let us consider A,B & C are three vague sets, then
Now to show, $A\backslash B = A \cap B' = A\backslash(A \cap B) = B'\backslash A'$

$t_{A\cap B'} = \min(t_A, t_{B'})$
$= \min(t_A, f_B) = t_{A\backslash B}$
$1 - f_{A\cap B'} = \min(1 - f_A, 1 - f_{B'})$
$= \min(1 - f_A, 1 - t_B) = 1 - f_{A\backslash B}$
Hence$A\backslash B = A \cap B'$

Again

$\Rightarrow t_{B'\backslash A'} = \min(t_{B'}, f_{A'})$
$= \min(f_B, t_A)$
$= \min(t_A, f_B)$
$= t_{A\backslash B}$
$\Rightarrow 1 - f_{B'\backslash A'} = \min(1 - f_{B'}, 1 - t_{A'})$
$= min(1 - t_B, 1 - f_A)$
$= min(1 - f_A, 1 - t_B) = 1 - f_{A\backslash B}$
Hence $B'\backslash A' = A\backslash B$

Again

$\Rightarrow t_{A\backslash(A\cap B)} = t_{(A\backslash A)\cup(A\backslash B)}, by\ using\ demorgan's\ law$
$= t_{\emptyset\cup(A\backslash B)}$
$= t_{A\backslash B}$
$1 - f_{A\backslash(A\cap B)} = 1 - f_{(A\backslash A)\cup(A\backslash B)} = 1 - f_{\emptyset\cup(A\backslash B)} = 1 - f_{A\backslash B}$
Hence the results are proved.

Now $t_{(A\cap B)\backslash C} = \min(t_{A\cap B}, f_C) = \min(\min(t_A, t_B), f_C)$
$= \min(t_A, min(t_B, f_C)) = min(t_A, t_{B\backslash C}) = t_{A\cap(B\backslash C)}$
$1 - f_{(A\cap B)\backslash C} = \min(1 - f_{A\cap B}, 1 - t_C) = \min(\min(1 - f_A, 1 - f_B), 1 - t_C)$
$= \min(1 - f_A, min(1 - f_B, 1 - t_c)) = \min(1 - f_A, 1 - f_{B\backslash C}) = 1 - f_{A\cap(B\backslash C)}$
Here we conclude $(A \cap B)\backslash C = A \cap (B\backslash C)$

$t_{A\backslash(B\cup C)} = \min(t_A, f_{B\cup C}) = \min(t_A, \min(f_B, f_C)$
$= \min(\min(t_A, f_B), f_C)) = \min(t_{A\backslash B}, f_C) = t_{(A\backslash B)\backslash C}$ $1 - f_{A\backslash(B\cup C)} = \min(1 - f_A, 1 - t_{B\cup C}) = \min(1 - f_A, 1 - \max(t_B, t_C))$
$= \min(1 - f_A, \min(1 - t_B, 1 - t_C)) = \min(\min(1 - f_A, 1 - t_B), 1 - t_c)$
$= \min(1 - f_{A\backslash B}, 1 - t_C) = 1 - f_{(A\backslash B)\backslash C}$
Here we proved that $A\backslash(B \cup C) = (A\backslash B)\backslash C$

Conclusion

In this paper, we have presented the notion of the difference on vague sets and establish some results. We have studies the different between the known generalization VS and IFS of fuzzy set. Vague sets are based on an interval based membership and that more expressive in capturing vagueness of data as the notion of truth membership function , false membership function and uncertainty function. That, understood in three aspects comprehensively: support degree, negative degree, and uncertainty degree. The notion of IFS and vague sets are regarded as equivalent, in sense that an IFS is isomorphic to a vague sets. When measuring vagueness in practice, we may use vague sets Is more natural than an IFS.

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