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A FIXED POINT THEOREM FOR TWO PAIRS OF OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS

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ABSTRACT

In this paper, we derive a common fixed point theorem for two pairs of occasionally weakly compatible mappings as an extension of fixed point theorem of a pair occasionally weakly compatible mapping established by Jungek and Rhoades.

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INTRODUCTION

Fuzzy set as a generalization of classical set was defined by L. A. Zadeh [7] in 1965 AD. Kramosil and Michalek [6] introduced concept of fuzzy metric space. Later, George and Veeramani [2] modified the notion of fuzzy metric space with help of continuous t-norms. Several researchers have derived fixed point theorems for fuzzy mappings on complete metric spaces. G. Jungek and B. E. Rhoades [4] defined compatibility and weakly compatibility of mappings. The concept of occasionally weakly compatible mapping was introduced by M. Al Thagafi and Naseer Shahzad [1] and then G. Jungek and B. E. Rhoades [4] proved a fixed point theorem for a pair of occasionally weakly compatible mappings. In this paper, we present a common fixed point theorem for two pairs of occasionally weakly compatible fuzzy mappings.

Definition 1.1[2]

A binary operation: $[0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $*$ satisfies following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a*1 = a$ for all □ $\in [0,1]$;
(iv) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$ for a, b, c, d $\in [0,1]$.

Definition 1.2[2]

A triplet $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions, for all $x, y, z \in X$,

(1) $M(x, y, 0) = 0$ and $M(x, y, t) > 0$ for all $t > 0$;
(2) $M(x, y, t) = 1$ *if and only if* $x = y$ for all $t > 0$;
(3) $M(x, y, t) = M(y, x, t) \neq 0$ for all $t > 0$;
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$ for s, $t > 0$;
(5) $M(x, y, .) : [0,\infty) \to [0,1]$ is left continuous and,
(6) $\lim_{t\to\infty} M(x,y,t) = 1$ for $t > 0$.

Then, M is called a fuzzy metric on X. Here, M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Definition 1.3[2]

Let $(X, M, *)$ be a fuzzy metric space. Then,

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a) sequence {xn} in X is said to converges to x in X if for each ε>0 and each t>0, there exists no ∈ N such that $M(x_n,x,t)>1-\varepsilon$ for all $n\geq n_o$ $(i.e.\ if \lim_{n\to\infty} M(x_n,x,t)=1\ for\ t>0)$.

(b) a sequence $\{x_n\}$ in X is said to be Cauchy if for each ε>0 and each t>0, there exists $n_o \in N$ such that $M(x_n,x_m,t)>1-\varepsilon$ for all $n,m\geq n_o$ $(i.e.\ if \lim_{n\to\infty} M(x_{n+p},x,t)=1\ for\ t>0, for p>0)$.

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete fuzzy metric space.

Definition 1.4[5]

(a) For any nonempty set X, a fuzzy set $\breve{A}$ is definedas set of ordered pairs $(x, A(x))$,where$A: X \to [0,1]$is called membership function and the collection of all fuzzy sets on X is denoted by$\mathcal{F}(X)$.

(b) A mapping F from X to $\mathcal{F}(Y)$ is called a fuzzy mapping if for each $x\in X$, $F(x)$ (sometimes denoted by F_x) is a fuzzy set on Y and $F_x(y)$ denotes the degree of membership of y in F_x

Definition 1.5[3]

Let X be a nonempty set and f,g selfmaps of X. A point x in X is called a coincidence point of f and $g$iff $fx=gx$. We shall call $w=fx=gx$ a point of coincidence of f and g.

1. Compatibility

Definition 2.1[3]

(a) Two self maps f and g of a fuzzy metric space $(X, M, *)$ are called compatible if $\lim_{n\to\infty} M(fgx_n,gfx_n,t)=1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

(b) Two self maps$f$ and g is called weakly compatible pair if they commute at coincidence points.

(c) Two self maps f and g of set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

A. Al-Thagafi and NaseerShahzad [1] have shown that weakly compatible is occasionally weakly compatible but converse is not true.

Example 2.2[1]

Let ℝ be the usual metric space.

Define F, G: ℝ→ℝ by Fx = 2x andGx = x2 for all x ∈ ℝ. Then, Fx = Gx for x = 0, 2 but FG0 = GF0 and FG2 ≠GF2. Maps F and G are occasionally weakly compatible self maps but not weakly compatible.

Lemma 2.3[1]

Let X be a nonempty set and f, g owc self maps of X. If f and g have a unique point of coincidence, $w = fx = gx$ then w is the unique common fixed point of f and g.

Now we deriveour common fixed point theorem for two pairs of occasionally weakly compatible (owc) mappings in fuzzy metric spaces.

2. Main Theorem

Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self-mapings of X. Let the pairs $\{A,S\}$ and $\{B,T\}$ be owc. If there exists $q\in(0, 1)$ such that

$$M(Ax, By, qt) \geq \min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

for all $x,y\in X$ and for all $t>0$, then there exists a unique point $w\in X$ such that $Aw=Sw=w$ and a unique point $z\in X$ such that $Bz=Tz=z$. Moreover, $z=w$, so that there is a unique common fixed point of A, B, S and T.

Proof

Let the pairs $\{A,S\}$ and $\{B,T\}$ be owc, so there are points

$x,y\in X$ such that $Ax=Sx$ and $By=Ty$. We claim that $Ax=By$.

If not, by inequality (1),

$M(Ax, By, qt) \geq min\ \{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t),$

$M(Ax, Ty, t), M(By, Sx, t)\} = min\ \{M(Ax, By, t), M(Ax, Ax, t), M(By, By, t),$

$M(Ax, By, t), M(By, Ax, t)\} = M(Ax, By, t)$.

Therefore $Ax = By$, i.e. $Ax = Sx = By = Ty$. Suppose that there isanother point z such that $Az = Sz$ then by (1), we have $Az = Sz = By = Ty$, so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma (2.3), w is the only common fixed point of A and S. Similarly there is a unique point $z\in X$ such that $z = Bz= Tz$.

Assume that $w \neq z$. We have

$M(w,z,qt) = M(Aw,Bz,qt)$

$\geq\min\ \{M(Sw,Tz,t), M(Sw,Aw,t), M(Bz,Tz,t), M(Aw,Tz,t), M(Bz,Sw,t)\}$

$= \min\ \{M(w,z,t), M(w,w,t), M(z,z,t), M(w,z,t), M(z,w,t)\}$

$= M(w,z,t)$.

Therefore, we have $z = w$ by Lemma (2.3). Hence, $z = w$ is a unique common fixed point of A, B, S and T.

REFERENCES

1. Al-Thagafi A. and Shahzad N., Generalised I-Nonexpansive self maps and invariant approximations, *Acta Mathematica*, 24 (5),[2008], pp. 867-876
2. George A. and Veeramani P., On some results in fuzzy metric spaces, Fuzzy sets and systems, 64,[1994], pp. 395-399
3. Jungek G. and Rhoades B. E., Fixed point theorems for occasionally weakly compatible mapping, Fixed point theory, 7(2), [2006], pp. 287-296
4. Jungek G., Compatible mappings and common fixed points, *International J. Math. Math. Sc.,* [1998], pp. 285-288
5. Klir, G.J. and Yuan, B., Fuzzy sets and fuzzy logic,[2013], PHI Pvt. Ltd.
6. Kramosil O. and Michalek J., Fuzzy metric and statistical metric spaces, Kybernetika, 11, [1975] pp. 326-334
7. Zadeh L. A., Fuzzy sets, information and control, 8, [1965], pp. 338-353

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