



Full Length Research Article

EFFECTS OF MAGNETIC FIELD WITH VARIABLE VISCOSITY AND N ENDOSCOPE ON PERISTALTIC MOTION

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ABSTRACT

The aim of the present investigation is to study the peristaltic transport through the gap between coaxial tubes, where the inner tube is an endoscope and the outer tube has a sinusoidal wave travelling down to its wall. The necessary theoretical results such as viscosity, pressure gradient and friction force on the inner and outer tubes have been obtained in terms of magnetic field. Out of these theoretical results the numerical solution of pressure gradient, outer friction, inert friction and flow rate are shown graphically for the better understanding of the problem.

INTRODUCTION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the esophagus, in urine transport from the kidney to the bladder through the urethra, in movement of chyme in the gastro-intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessel such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces. A number of analytical (Shapiro *et al.*, 1969; Zien and Ostrach, 1970; Elshehawey and Mekheimer, 1994; Ramachandra and Usha, 1995; Mekheimer Elsayed *et al.*, 1998; Mekhemier, 2003 & 2002), numerical and experimental (Takabatake and Ayukawa, 1982 & 1988; Tang and Shen, 1993; Brown and Hung, 1977; Latham, 1966), studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava (Srivastava and Srivastava, 1984), and some imported contribution of recent year, are reference in Srivastava and Saxsen (Srivastava and Saxena, 1994). Physiological organs are generally observed have the form of a non-uniform duct (Lee and Hung, 1971; Wiedeman, 1963). In particular, the vas deferens in rhesus monkey is in the form of a diverging tube with a ration of exit to inlet dimensions of approximately four (Guh et al., 1975). Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Recently, Srivastava *et al.*, (1983) and Srivastava and Srivastava (Srivastava and Srivastava, 1988) studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. In the view of above discussion the effect of magnetic fluid with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave travelling down to its wall is the aim of present investigation.

Formulation and analysis

Consider the two-dimensional flow of an incompressible Newtonian fluid with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave travelling down its wall. The geometry of the two-wall surface is given by the equation:

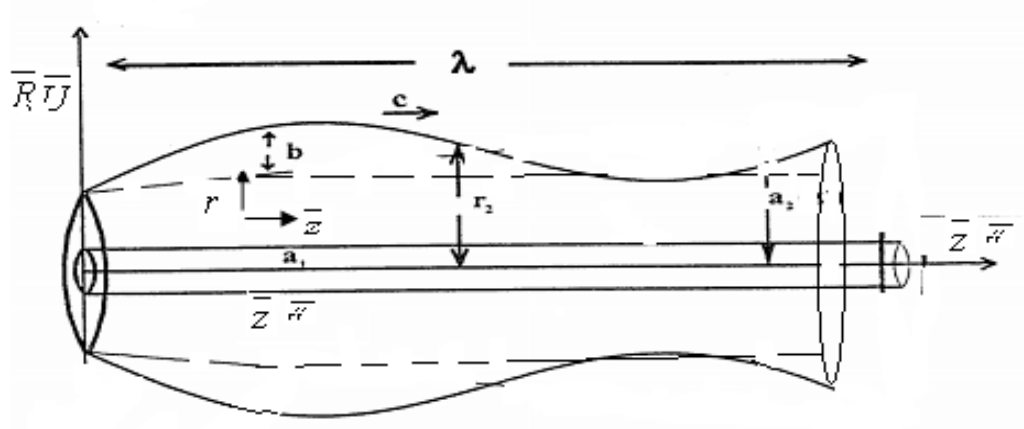
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$$\bar{r}_1 = a_1, \quad \dots \dots \dots (2.1)$$

$$\bar{r}_1 = a_1 + b \sin \frac{2\pi}{\lambda} (\bar{z} - c\bar{t}) \quad \dots \dots \dots (2.2)$$

Where a_1 is the radius of endoscope a_2 is the radius of the small intestine at inlet, b is the amplitude of the wave, λ is the wavelength, $\bar{t}$ is time and c is the wave speed.



In the fixed coordinates $(\bar{r}, \bar{z})$ the flow in the gap between inner and outer tubes is unsteady but if we choose moving coordinates $(\bar{r}, \bar{z})$ which travel in the $\bar{z}$ -direction with the same speed as the wave, then the flow can be treated as steady. The coordinate's frames are related through:

$$\bar{z} = \bar{Z} - c\bar{t}, \quad \bar{r} = \bar{R}, \quad \dots \dots \dots (2.3)$$

$$\bar{w} = \bar{W} - c, \quad \bar{u} = \bar{U}, \quad \dots \dots \dots (2.4)$$

Where $\bar{u}, \bar{w}$ and $\bar{u}, \bar{w}$ are the velocity components in the radial and axial direction in the fixed and moving coordinates respectively. Equations of boundary condition in the moving coordinates is

$$\frac{1}{\bar{r}} \frac{\partial (\bar{r} \bar{u})}{\partial \bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \quad \dots \dots \dots (2.5)$$

And the Navier Stokes equation

$$\begin{aligned} \rho (\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}}) = & - \frac{\partial \bar{p}}{\partial \bar{r}} + \frac{\partial}{\partial \bar{r}} [2\bar{\mu}(\bar{r}) \frac{\partial \bar{u}}{\partial \bar{r}}] + \frac{2\bar{\mu}(\bar{r})}{\bar{r}} (\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}}) \\ & + \frac{\partial}{\partial \bar{z}} [\bar{\mu}(\bar{r}) (\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}})] - \sigma B_0^2 (\bar{u}) \end{aligned} \quad \dots \dots \dots (2.6)$$

$$\begin{aligned} \rho (\bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}}) = & - \frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}} [2\bar{\mu}(\bar{r}) \frac{\partial \bar{w}}{\partial \bar{z}}] + \\ & \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} [\bar{\mu}(\bar{r}) \bar{r} (\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}})] - \sigma B_0^2 (\bar{w}) \end{aligned} \quad \dots \dots \dots (2.7)$$

$\bar{p}$ is the pressure, $\bar{\mu}(\bar{r})$ is the viscosity function, σ is Electric conductivity and B_0 is applied magnetic field. The boundary condition are written

$$\bar{w} = -c \quad \text{at} \quad \bar{r} = \bar{r}_1 \quad \bar{r} = \bar{r}_2 \quad \dots \dots \dots (2.8a)$$

$$\bar{u} = 0 \quad \text{at} \quad \bar{r} = \bar{r}_1 \quad \dots\dots\dots (2.8b)$$

Introducing the non-dimensional variable, the Reynolds number (Re), and the wave number (δ) as follows:

$$r = \frac{\bar{r}}{a_{20}}, \quad R = \frac{\bar{R}}{a_{20}}, \quad r_1 = \frac{\bar{r}_1}{a_{20}} = \frac{a_{10}}{a_{20}} = \varepsilon < 1, \quad z = \frac{\bar{z}}{\lambda}, \quad Z = \frac{\bar{Z}}{\lambda}, \quad \mu(r) = \frac{\bar{\mu}(\bar{r})}{\mu_0},$$

$$u = \frac{\lambda \bar{u}}{a_{20} c}, \quad U = \frac{\lambda \bar{U}}{a_{20} c}, \quad w = \frac{\bar{w}}{c}, \quad W = \frac{\bar{W}}{c}, \quad \delta = \frac{a_{20}}{\lambda} \ll 1,$$

$$\text{Re} = \frac{c a_{20} \rho}{\mu_0}, \quad p = \frac{a_{20}^2 \bar{p}}{c \lambda \mu_0}, \quad t = \frac{c \bar{t}}{\lambda}, \quad \phi = \frac{b}{a_{20}} < 1,$$

$$r_2 = \frac{\bar{r}_2}{a_{20}} = 1 + \phi \sin 2\pi z \quad \dots\dots\dots (2.9)$$

where ε is the radius ratio, ϕ is the amplitude ratio and μ_0 is the viscosity on the endoscope. Equation of motion and boundary conditions in the dimensionless form become:

$$\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad \dots\dots\dots (2.10)$$

$$\begin{aligned} \text{Re} \delta^3 \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) &= -\frac{\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} \left(2\mu(r) \frac{\partial u}{\partial r} \right) + \delta^2 \frac{\partial}{\partial z} \left[\mu(r) \left(\delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right. \\ &\quad \left. + \frac{2\delta^2 \mu(r)}{r} \left(\frac{\partial u}{\partial r} - \frac{u}{r} \right) - \delta^2 M^2(u) \right] \quad \dots\dots\dots (2.11) \end{aligned}$$

$$\begin{aligned} \text{Re} \delta \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[\mu(r) r \left(\delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right] \\ &\quad + \delta^2 \frac{\partial}{\partial z} \left(2\mu(r) \frac{\partial w}{\partial z} \right) - M^2(w) \quad \dots\dots\dots (2.12) \end{aligned}$$

$$M = \sqrt{\frac{\sigma}{\mu}} B_0 a_{20} \text{ is Hartmann number, } \sigma \text{ is Electric conductivity}$$

With the dimensionless boundary condition

$$w = -1 \quad \text{at} \quad r = r_1, \quad r = r_2 \quad \dots\dots\dots (2.13a)$$

$$u = 0 \quad \text{at} \quad r = r_1. \quad \dots\dots\dots (2.13b)$$

Using the long wavelength approximation and neglecting the wave number ($\delta = 0$), one can reduce eqs. (6.2.10) and (6.2.12)

$$\frac{\partial p}{\partial r} = 0 \quad \dots\dots\dots (2.14)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(\mu(r) r \frac{\partial w}{\partial r} \right) - M^2(w) \quad \dots\dots\dots (2.15)$$

The instantaneous volume flow rate in the fixed coordinate system is given by:

$$\bar{Q} = 2\pi \int_{\bar{r}_1}^{\bar{r}_2} \bar{W} \bar{R} d\bar{R} \quad \dots\dots\dots (2.16)$$

Where $\bar{r}_1$ is a constant and $\bar{r}_2$ is a function of $\bar{Z}$ and $\bar{t}$. On substituting eqs. (2.3) and (2.4) into eqs. (6.2.16) and the integrating, one obtains:

$$\bar{Q} = \bar{q} + \pi c(\bar{r}_2^2 - \bar{r}_1^2) \quad \dots\dots\dots (2.17)$$

Where

$$\bar{q} = 2\pi \int_{\bar{r}_1}^{\bar{r}_2} \bar{w} \bar{r} d\bar{r} \quad \dots\dots\dots (2.18)$$

is the volume flow rate in the moving coordinate system and is independent of time. Here, $\bar{r}_2$ is a function of $\bar{z}$ alone and is defined through equ. (2.2). Using the dimensionless variable, eq. (2.18) becomes

$$F = \frac{q}{2\pi a_2^2 c} = \int_{r_1}^{r_2} w r dr \quad \dots\dots\dots (2.19)$$

The time- mean flow over a period $T = \frac{\lambda}{c}$ at a fixed Z position is defined as:

$$Q = \frac{1}{T} \int_0^T \bar{Q} d\bar{t} \quad \dots\dots\dots (2.20)$$

Using eqs. (2.17) and (2.18) in eq. (2.20) and integrating, we get:

$$\bar{Q} = \bar{q} + \pi c(a_2^2 - a_1^2 + \frac{b^2}{2})$$

Which may be written as:

$$\frac{\bar{Q}}{2\pi a_{20}^2 c} = \frac{\bar{q}}{2\pi a_{20}^2 c} + \frac{1}{2}(1 - \varepsilon^2 + \frac{\phi^2}{2}) \quad \dots\dots\dots (2.21)$$

On defining the dimensionless time-mean flow as:

$$\Theta = \frac{\bar{Q}}{2\pi a_{20}^2 c}$$

Writing the eq. (6.2.21) as:

$$\Theta = F + \frac{1}{2}(1 - \varepsilon^2 + \frac{\phi^2}{2}) \quad \dots\dots\dots (2.22)$$

Solving eqs. (2.13) - (2.15) we obtain:

$$W = \frac{1}{2} \frac{dp}{dz} \left[I_1(r) - \frac{I_2(r)\{I_1(r_2) - I_1(r_1)\} - I_1(r_1)I_2(r_2) - I_2(r_1)I_1(r_2)}{I_2(r_2) - I_2(r_1)} \right] - (1/2)M^2(D) - 1 \quad \dots\dots\dots (2.23)$$

$$\text{Where } D = \frac{I_2(r)\{I_1(r_2) - I_1(r_1)\} - I_1(r_1)I_2(r_2) - I_2(r_1)I_1(r_2)}{I_2(r_2) - I_2(r_1)}$$

$$I_1(r) = \int \frac{r}{\mu(r)} dr \quad \dots\dots\dots (2.24)$$

$$I_2(r) = \int \frac{dr}{r\mu(r)} \dots\dots\dots (2.25)$$

Using eq. (2.19), obtain the relationship between $\frac{dp}{dz}$ and F as follows:

$$F = \frac{1}{4} \frac{dp}{dz} \left[\frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3 \right] + \frac{1}{4} M^2 \left[\frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3 \right] - \frac{1}{2} (r_2^2 - r_1^2) \dots\dots\dots (2.26)$$

$$I_3 = \int_{r_1}^{r_2} \frac{r^3}{\mu(r)} dr \dots\dots\dots (2.27)$$

Solving eq. (2.26) for $\frac{dp}{dz}$, we obtain:

$$\frac{dp}{dz} = \frac{4F + 2(r_2^2 - r_1^2) - (1/4)M^2 \left(\frac{[I_1(r_2) - I_1(r_1)]^2}{I_2(r_2) - I_2(r_1)} - I_3 \right)}{\frac{[I_1(r_2) - I_1(r_1)]^2}{I_2(r_2) - I_2(r_1)} - I_3} \dots\dots\dots (2.28)$$

The pressure rise ΔP_λ and friction force on inner and outer tubes $F_\lambda^{(i)}$ and $F_\lambda^{(o)}$, in their non-dimensional forms, are given by:

$$\Delta P_\lambda = \int_0^1 \left(\frac{dp}{dz} \right) dz \dots\dots\dots (2.29)$$

$$F_\lambda^{(i)} = \int_0^1 r_1^2 \left(- \frac{dp}{dz} \right) dz \dots\dots\dots (2.30)$$

$$F_\lambda^{(o)} = \int_0^1 r_2^2 \left(- \frac{dp}{dz} \right) dz \dots\dots\dots (2.31)$$

The effect of viscosity variation on peristaltic transport can be investigated through eq. (2.29) - (2.31) for any given viscosity function $\mu(r)$.

For the present instigation, assume that the viscosity variation in the dimensionless form followed by Srivastava *et al.* (1984) is given by:

$$\mu(r) = e^{-\alpha r} \dots\dots\dots (2.32)$$

Or

$$\mu(r) = 1 - \alpha r \text{ for } \alpha \ll 1 \dots\dots\dots (2.33)$$

where α is viscosity parameter. The assumption is reasonable for the following physiological reason. Since normal person or animal or similar size takes 1 to 2L of fluid every day. On the fact of that, another 6 to 7L of fluid received by the small intestine daily as secretion from salivary glands, stomach, pancreas, liver and the small intestine itself. This implies that concentration of fluid is dependent on the radial distance. Therefore, the above choice of $\mu(r) = e^{-\alpha r}$ is justified.

Substituting eq. (2.33) into eqs. (2.24), (2.25) and (2.27), and using eq. (2.28), we obtain:

$$\frac{dp}{dz} = \left[\left(\{ 16\Theta - 8(1 - \epsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2) \} \right) / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right] \times (1 - (4\alpha + M^2) \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3 \log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2(r_2 - r_1)}{4(\ln(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\})$$

$$\left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right\} \dots\dots\dots (2.34)$$

Substituting eq.(2.34) in eq.(2.29)-(2.31) yield:

$$\Delta P_\lambda = \int_0^1 \left[\left(\{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right) \right. \right. \\ \left. \times (1 - (4\alpha + M^2) \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3 \log(r_2 / r_1)} \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right\} \right) \right] dz \dots\dots\dots (2.35)$$

$$F_\lambda^{(i)} = \int_0^1 -r_1^2 \left[\left(\{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right) \right. \right. \\ \left. \times (1 - (4\alpha + M^2) \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3 \log(r_2 / r_1)} \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right\} \right) \right] dz \dots\dots\dots (2.36)$$

$$F_\lambda^{(o)} = \int_0^1 -r_2^2 \left[\left(\{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right) \right. \right. \\ \left. \times (1 - (4\alpha + M^2) \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3 \log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2 (r_2 - r_1)}{4(\log(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\} \right. \right. \\ \left. \left. \left/ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right\} \right) \right] dz \dots\dots\dots (2.37)$$

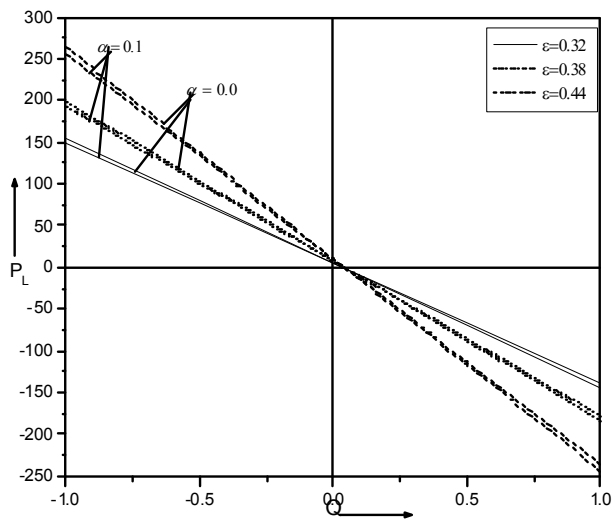
RESULTS, DISCUSSIONS AND CONCLUSION

The dimensionless pressure rise (P_λ) and the friction forces on the inner and outer tube for different given values of the dimensionless flow rate Θ , amplitude ratio ϕ , radius ratio ε , Hartmann number M and viscosity parameter α are computed using the equation (2.35) to (6.2.37). As the integrals in equation. (2.35) to (2.37) are not integrable in the closed form so they are evaluated using $a_{20}=1.25\text{cm}$ $a/\lambda=0.156$

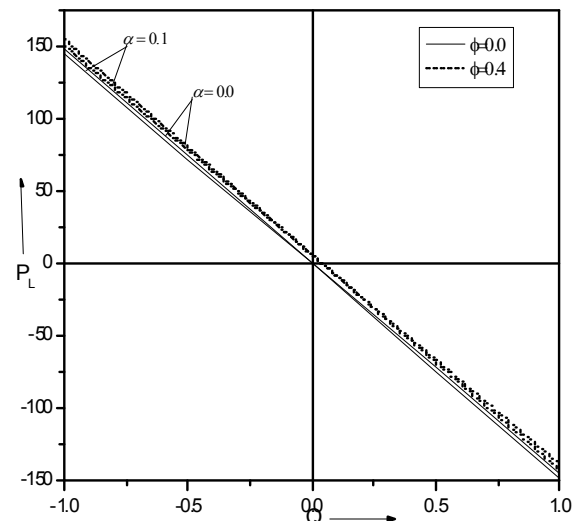
The values of viscosity parameter α as reported by Srivastava *et al.* (1984) are $\alpha=0.0$, $\alpha=0.1$ Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported by Cotton and Williams [17] and radius ratio 1.25cm reported by Srivastava and Srivastava (1984). Fig. (1) Shows the pressure rise against the flow rate here it is observed that the pressure increases with the increase of flow rate for different values of radius ratio $\varepsilon=0.32, \varepsilon=0.38$ and $\varepsilon=0.44$ and pressure decreases for the viscosity $\alpha=0.0$ and $\alpha=0.1$. Fig (2) shows that as the viscosity α increases the pressure is decreases. And for the different values of amplitude ratio $\phi=0.0$ and $\phi=0.4$ the pressure is decreases.

Fig (3) and (4) shows the friction force on the outer tube for different values of radius ratio and amplitude ratio, here it is observed that as radius ratio increases the friction force also decreases and they are independent of radius ratio at certain values of the flow rate [for the values $\phi=0.4$ and $\alpha=0.0$ and $\alpha=0.1$].

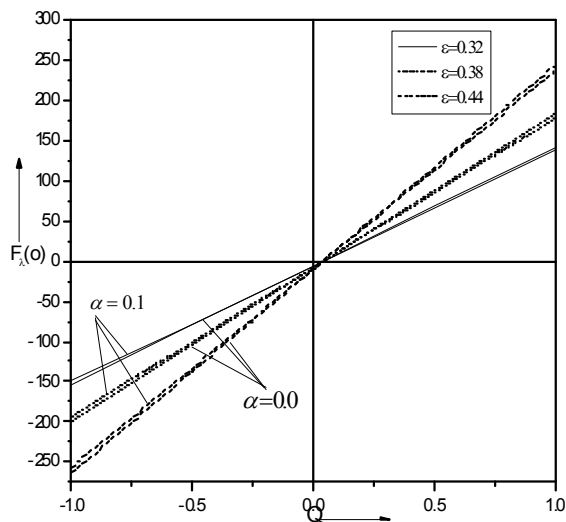
In fig (5) and (6) it is noticed that the friction force on the inner tube (endoscope) and on outer tube is plotted against the flow rate for different values of amplitude ratio ϕ and for different values radius ratio $\varepsilon=0.32, \varepsilon=0.38$ and $\varepsilon=0.44$ and for the values of viscosity $\alpha=0.0$ and $\alpha=0.1$. It is noticed that as the amplitude ratio ϕ increases the friction force on the outer tube and inner tube decreases and as the viscosity increases the friction force on the outer tube and inner tube decreases. From Fig (7) it is noticed that the pressure rise increases for different values of applies magnetic field $M=1, M=3, M=5$. In (8) and (9) it is noticed that the friction force decreases on endoscope and on the outer tube as magnetic field $M=1, M=3, M=5$ increases.



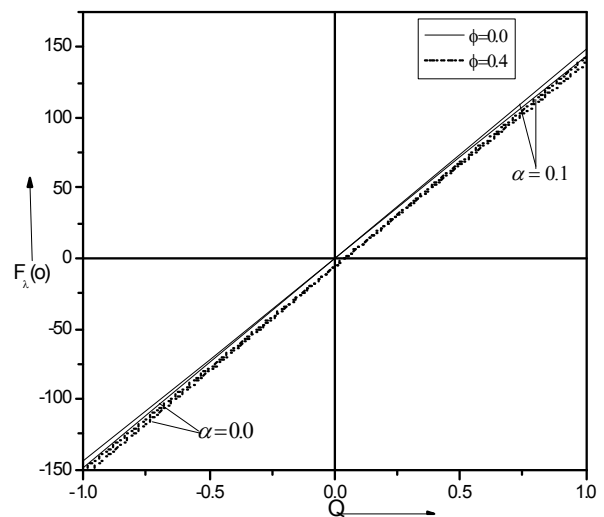
Fig(1) The pressure rise versus flow rate for $\phi=0.4$ and $\alpha=0.0$ and $\alpha=0.1$



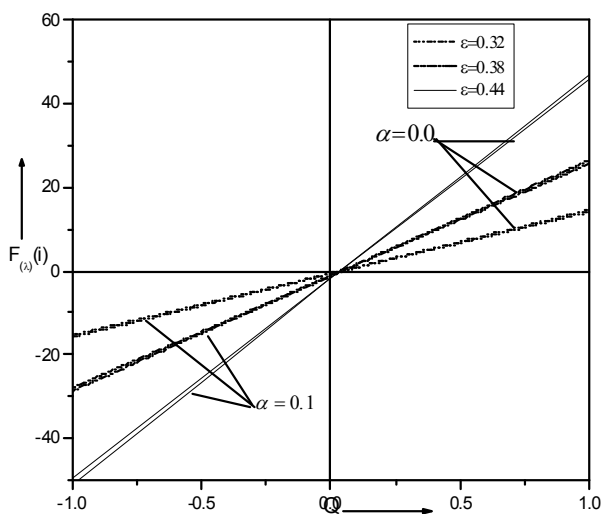
Fig(2) The pressure rise versus flow rate for $\varepsilon=0.32$, $\alpha=0.0$ and $\alpha=0.1$



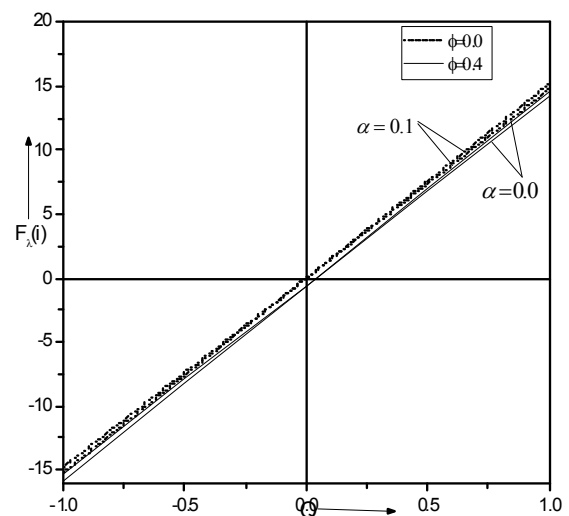
Fig(3) The friction force on the outer tube versus flow rate for $\phi=0.2$ and $\alpha=0.0$, $\alpha=0.1$



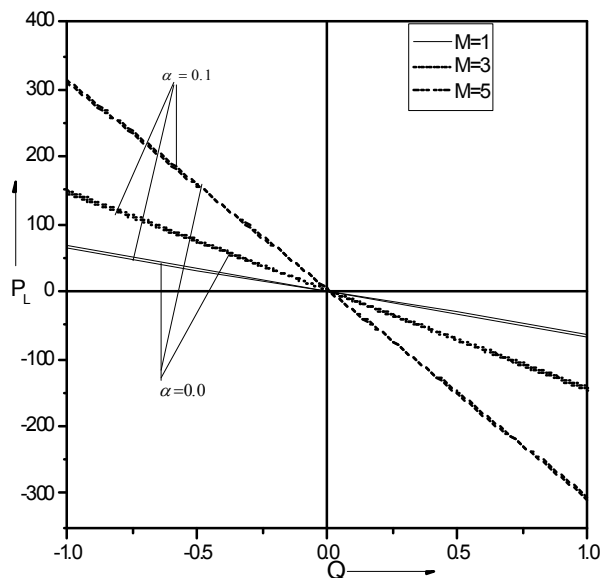
Fig(4) The friction force on the outer tube versus flow rate for $\varepsilon=0.32$, $\alpha=0.0$, $\alpha=0.1$



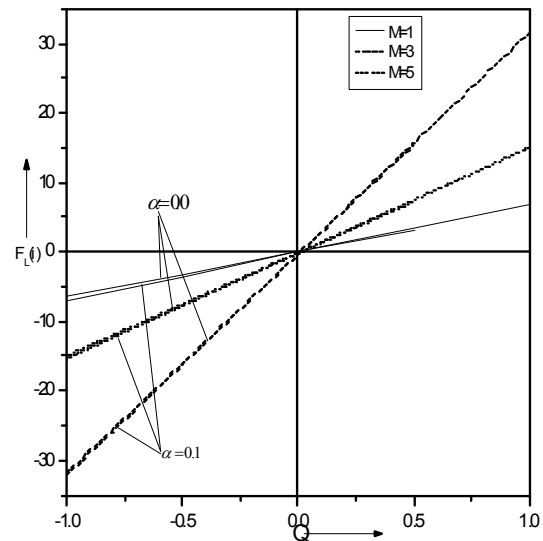
Fig(5) The friction on the inner tube (endoscope) versus flow rate for $\phi=0.2$, $\alpha=0.0$, $\alpha=0.1$



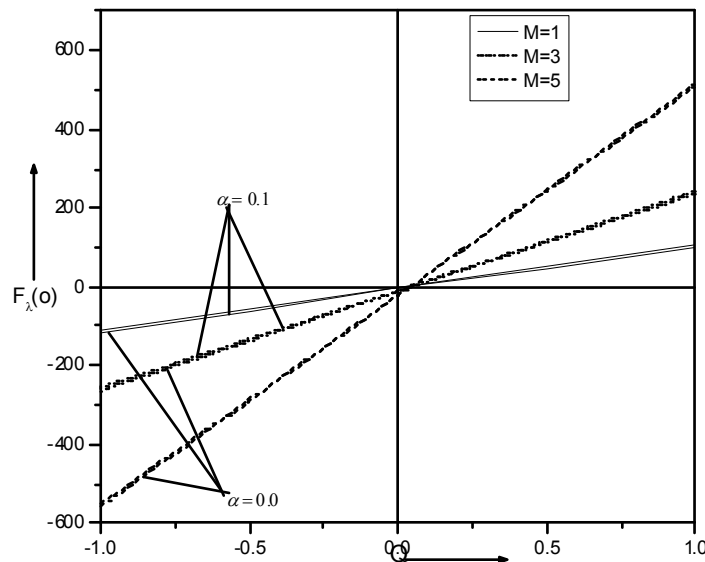
Fig(6) The friction on the inner tube (endoscope) versus flow rate for $\varepsilon=0.32$ and $\alpha=0.0$, $\alpha=0.1$



Fig(7) The pressure versus flow rate for $\varepsilon=0.32$, $\alpha=0.0$, $\alpha=0.1$ and $\phi=0.2$



Fig(8) The friction on the inner tube (endoscope) versus flow rate for different magnetic field for $\phi=0.2$, $\varepsilon=0.32$, $\alpha=0.0$ and $\alpha=0.1$



Fig(9) The friction on the outer tube versus flow rate for different magnetic field for $\varepsilon=0.32$, $\phi=0.2$, $\alpha=0.0$ and $\alpha=0.1$

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